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COMMENTS ON POWER SPECTRA OF DISCRETE STOCHASTIC TIME SERIES*

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DISCLAIMER

COMMENTS ON POWER SPECTRA OF DISCRETE STOCHASTIC TIME SERIES*

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ABSTRACT

We show that the slope of slightly flatter than −2 seen in the power spectra of stochastic series is a consequence of finite discrete systems observed with limited temporal correlation.

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In lattice models, one describes nature at the microscopic level in terms of a discrete space time, and then obtains a macroscopic level description by coarse graining.[1] In this note, we wish to point out that coarse graining itself implies the power law spectra often measured for stochastic systems. Furthermore, the resultant exponent of slightly flatter than −2 is a signature of a discrete rather than continuous system.

To fix our ideas, consider a stochastic time series $f_j = f(t_j)$, j=1 to N, with discrete Fourier transform $g(\omega)$:

$$g(\omega) = \sum_j f_j \exp(i\omega t_j) \tag{1}$$

Here $|g(\omega)|^2$ is the power spectrum; its ensemble average is independent of frequency ω for a white noise source. We attempt to measure this process on the macroscopic level with a physical device of limited temporal resolution. Suppose the impulse response of the device is W(t), then the measured signal will be F(t) = f(t)*W(t) with power spectrum $|F(\omega)|^2 = |g(\omega)|^2 |V(\omega)|^2$. Here, the star denotes convolution and $V(\omega)$ is the Fourier transform of the impulse response W(t).

The simplest possible case is that of a running average over a time interval T ie: W(t) = 1 for $0 < t < T$ and 0 otherwise. In this case, the measured power spectrum is proportional to

$$|V(\omega)|^2 = (\sin \omega T/2 \;/\; \sin \omega\Delta t/2)^2 \tag{2}$$

This function is plotted in Fig. 1 for N=2048 and a window of T=200 Δt. It is seen that the spectrum displays an apparent power law behavior in the high frequency range with exponent slightly flatter than −2. The apparent leveling at the highest ω is due to the finite spectral width and periodicity of the finite Fourier transform. The approximate slope of -1.7 is nearly independent of the size of the window as long as the window T is small relative to N. Physically, we can think of T as a correlation time. Then statistically relevant results require $T << N\Delta t$. The continuum limit should be approached when $\Delta t << T << N\Delta t$. For a smaller T, the effect of discreteness will be manifested.

This result can be understood by taking the logarithmic derivative $d\ell n|V(\omega)|^2/d\ell n\ \omega$. The slowly varying envelope seen in Fig. 1 is due to the denomination of Eq. 2. The resulting slope varies from -2 to about -1.56 as $\omega\Delta t$ varies over the physically meaningful range; the average value is -1.86. This result is very suggestive in light of experimentally determined power spectra of turbulent systems on the one hand, and fluctuations near a critical point on the other (the critical exponent for the correlation function is $-2 + \eta$ where $\eta \approx .14$ experimentally).

As a demonstration of these ideas, we constructed a finite stochastic time series of $N = 2048$ values with a random number generator. The power spectrum of the series was found to be flat as expected. The "measured" time series was generated by using a running average of width $200\ \Delta t$. The trend can be made more evident by performing a running average in frequency as well; this has been done in Fig. 2 over a frequency width of $20\ \Delta\omega$. The slope of the spectrum is consistent with -1.7.

We conclude that the slope of -2 is characteristic of a <u>discrete</u> process with limited time correlation, in distinction to the slope of -2 characteristic of a continuum.[2] Furthermore, the power law spectrum is a natural consequence of <u>finite</u> spatial or temporal resolution measurements of <u>finite</u> time series. The imposition of correlation through finite resolution observation is enough to lead to power law spectra.

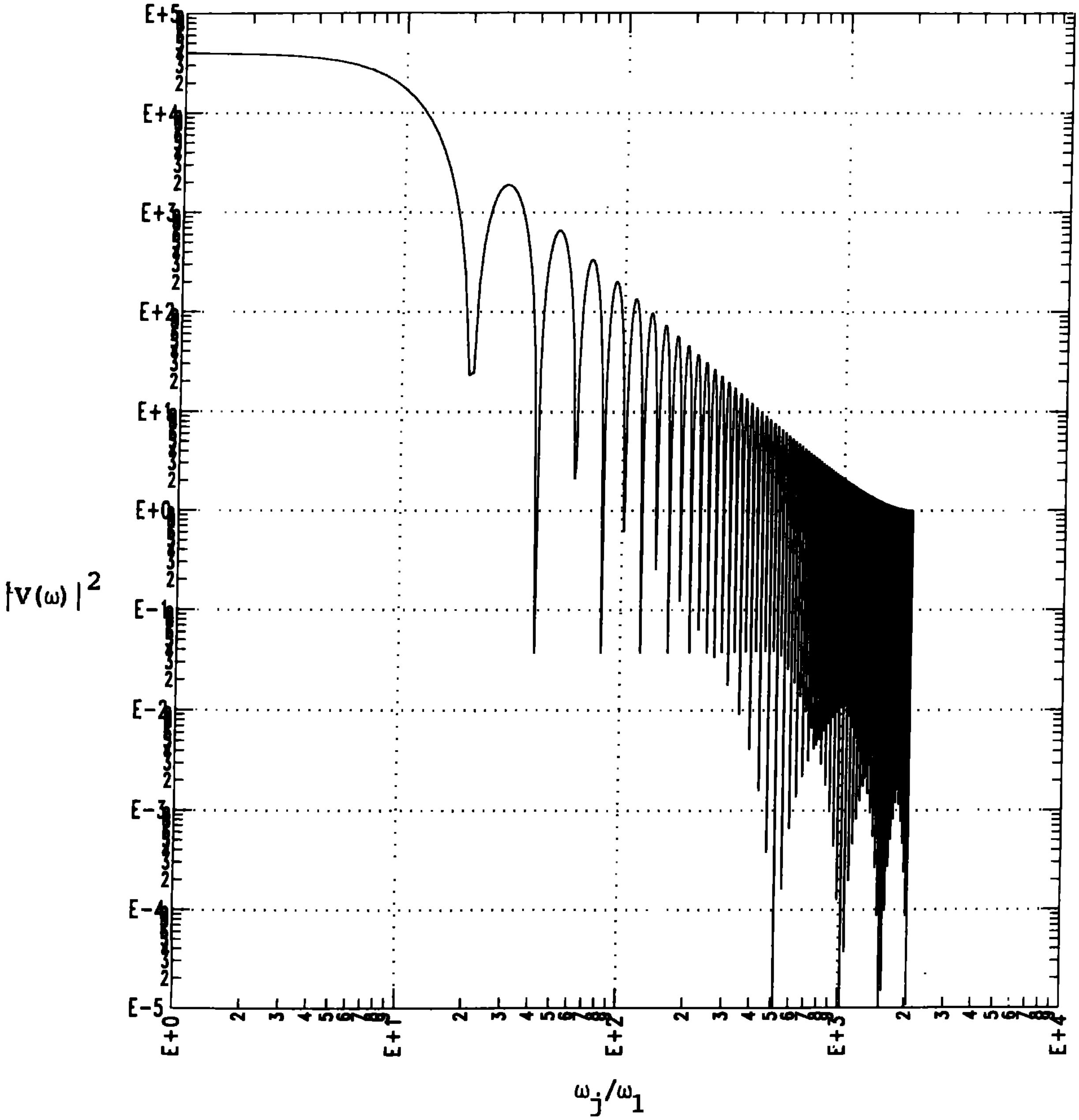

Fig.1

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2. For the continuum case, Eq. 2 becomes $|V(\omega)|^2 = (\sin \omega\, T/2)^2/(\omega/2)^2$.

FIGURE CAPTIONS

Fig. 1 Plot of $|V(\omega)|^2$ given by Eq. 2 for a time series with N = 4096 and T = 200 Δt.

Fig. 2 The power spectrum of a time series of N = 4096 random numbers with T = 200 Δt and a averaging frequency window of 20 $\Delta\omega$.

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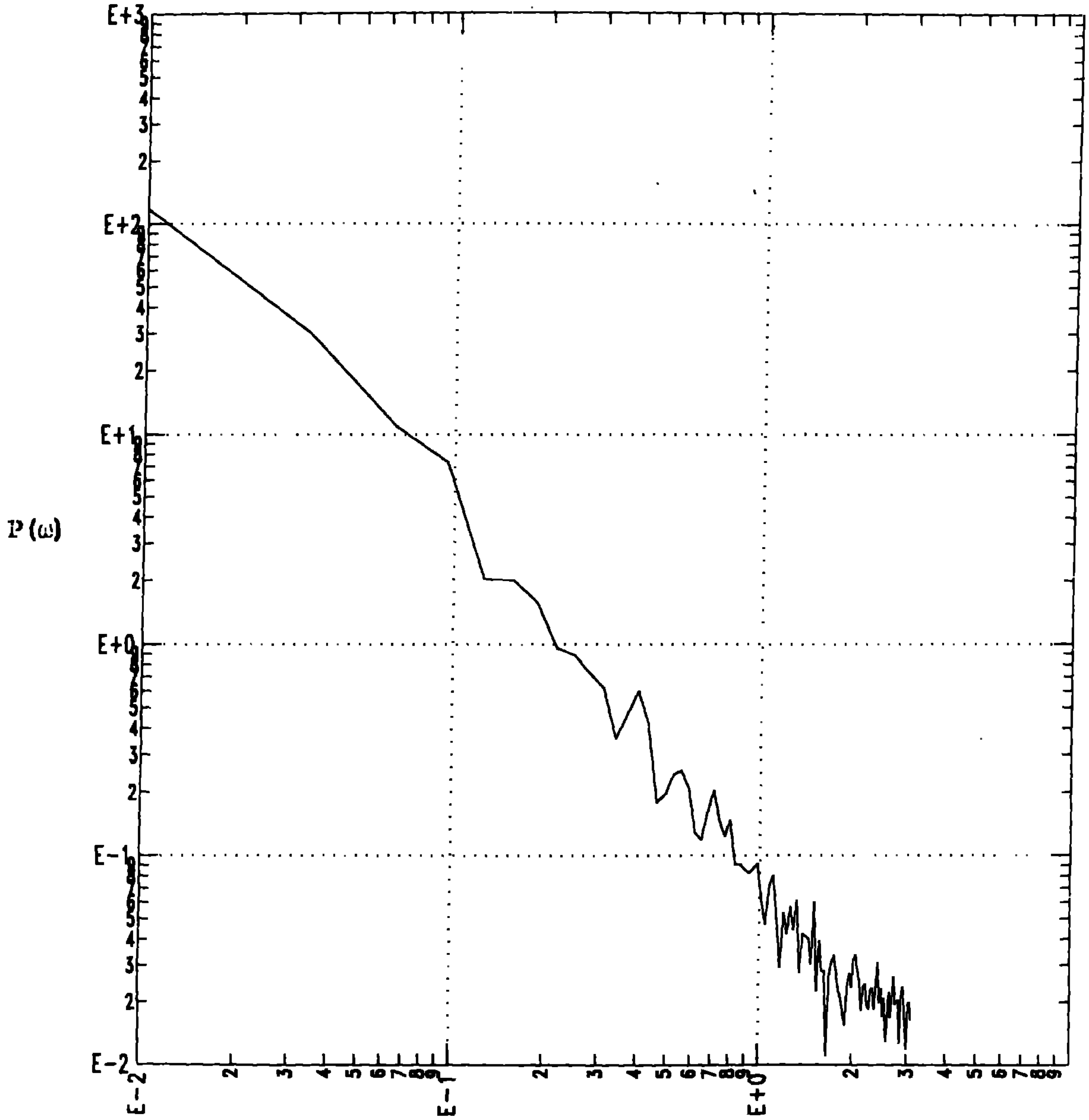

Fig. 2

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